

Modeling complex state of block-layered rock mass with hierarchic inclusions

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Abstract. Iterative algorithms are constructed for 2D modeling of diffraction of sound and linearly polarized S-wave on an inclusion having a hierarchical elastic structure in a J-th layer of an N-layer elastic medium. Under analysis are the cases when the inclusion of each rank and the enclosing medium have different densities and the same elastic parameters and when they have the same densities and different elastic parameters. The space–frequency representation of the wave field distribution uses the methods of the integral and integral–differential equations.

1. Introduction

Using 3D electromagnetic induction space-and-time monitoring, the authors [1, 2] illustrated applicability of the hierarchical discrete medium model to describing structure of rock mass composed of various substances [3]. In the framework of a specific modification of the model, two hierarchical levels were analyzed. Disintegration zones [4] around a tunnel occur asymmetrically at the tunnel roof and floor, which may be an evidence of the nonequilibrium state of the medium. These zones occur discretely, i.e. they are absent in certain intervals in rock mass surrounding tunnels. The maximum change in rock mass exposed to mining impact is the change in the morphology of spatial layout of these zones. The obtained results make it possible to conclude that a rock mass is a multi-rank hierarchical structure. The rock mass dynamics, structure and self-organization can be studied using the flatbed multi-level electromagnetic induction method with a controllable source and the appropriate data processing and interpretation procedure to reveal disintegration zones—indicators of rock mass stability.

Currently the results of theoretical modeling of electromagnetic and seismic field in a layered medium with inclusions of hierarchical structure are commonly used in construction of new systems for integrated monitoring of complex structure media [5]. The modeling algorithm for 3D and 2D nonuniformities in electromagnetic and seismic cases, respectively, are constructed [6–8]. It is shown that as the hierarchical pattern of a medium becomes more complex, the degree of spatial nonlinearity in seismic and electromagnetic fields grows, which agrees with the detailed monitoring tests in rockburst-hazardous Tashtagol and North-Ural Bauxite Mines. The developed theory demonstrates complication of the process of complexing methods using electromagnetic and seismic fields to study response of medium with a hierarchical structure. This problem is inseparable from the formulation and solving of an inverse problem of propagating electromagnetic and seismic fields in such complex media. The problem of the inverse problem algorithm using the equations of the theoretical inverse



problem for the 2D Helmholtz equation is discussed in [9, 10]. The explicit equation of the theoretical inverse problem are written for the cases of the electromagnetic field dissipation (polarization E and H) and the linearly polarized elastic wave dissipation in a layered conductive medium and in an elastic medium having a hierarchical conductive or elastic inclusion, which are the framework for delineating non-coaxial inclusions of the first rank of the hierarchical structure. Evidently, the inverse problem solution uses the input data of monitoring adjusted to hierarchical structure of a medium. On the other hand, in a more complex medium, each wave field brings information on the internal structure and condition of the medium, and the interpretation of the seismic and electromagnetic fields should be carried out separately. This study addresses the issue of complexing of different-type geophysical fields propagating in hierarchically structured media.

2. Modeling sound diffraction on 2D hierarchical nonuniformity of anomalous density in an N-layer elastic medium

The work [11] describes the modeling algorithm for the sound diffraction in on 2D elastic hierarchical inclusion in a J-th layer of an N-layer medium. In this case, $k_{1ji}^2 = \omega^2(\sigma_{ji} / \lambda_{ji})$ —P-wave number, where ji indicate properties inside the nonuniformity while ja —outside the nonuniformity, λ —Lame constant, σ —density; ω —angular rate, $\vec{u} = \text{grad}\varphi$ —vector of displacements, φ^0 —potential of the normal seismic field in the layered medium without nonuniformity: $\varphi_{ji}^0 = \varphi_{ja}^0$. It is assumed that elastic characteristics of the hierarchical inclusion are similar for all ranks 1 and enclosing layer, while its density differs at all ranks from the density of the enclosing medium; then the system of equations [3] is rewritten as:

$$\begin{aligned} & \frac{(k_{1ji}^2 - k_{1j}^2)}{2\pi} \iint_{S_{cl}} \varphi_l(M) G_{Sp,j}(M, M^0) d\tau_M + \frac{\sigma_{ja}}{\sigma_{ji}} \varphi_{l-1}^0(M^0) - \\ & - \frac{(\sigma_{ja} - \sigma_{jil})}{\sigma_{jil} 2\pi} \oint_{Cl} G_{Sp,j} \frac{\partial \varphi_l}{\partial n} dc = \varphi(M^0), M^0 \in S_{Cl}, \\ & \frac{\sigma_{jil}(k_{1jil}^2 - k_{1j}^2)}{\sigma(M^0) 2\pi} \iint_{S_{cl}} \varphi_l(M) G_{Sp,j}(M, M^0) d\tau_M + \varphi_{l-1}^0(M^0) - \\ & - \frac{(\sigma_{ja} - \sigma_{jil})}{\sigma(M^0) 2\pi} \oint_{Cl} G_{Sp,j} \frac{\partial \varphi_l}{\partial n} dc = \varphi_l(M^0), M^0 \notin S_{Cl}, \end{aligned} \quad (1)$$

where $k_{1ji}^2 = \omega^2(\sigma_{ji} / \lambda_{ji})$ —P-wave number, where ji indicate properties inside the nonuniformity while ja —outside the nonuniformity, $l = 1 \dots L$ —hierarchical level number, φ_l^0 — potential of the normal seismic field in the layered medium when there is no nonuniformity of the previous rank: $\varphi_l^0 = \varphi_{l-1}$ when $l = 2 \dots L$ and $\varphi_l^0 = \varphi^0$ when $l = 1$, which conforms with the corresponding expression [11].

Given that the transition to the next hierarchical level causes no changes in the two-dimensionality index and only the geometry of cross-sections of the nested structures is changed, it is possible to describe the iterative process of acoustic field modeling (the case of formation of P-wave only). The iterative process relates with the modeling the displacement vector during transition to the next hierarchical level. Inside each hierarchical level, the integral-differential equation and integral-differential representation are calculated using by the algorithm (1). When the local nonuniformity structure divides into a number of nonuniformities at a certain hierarchical level, the integrals in the in (1) are taken around all non-uniformities.

3. Modeling diffraction of elastic S-wave on hierarchical nonuniformity of anomalous density in an N -layer elastic medium

Similarly to (1) modeling of an elastic S-wave in an N -layer medium having 2D structural hierarchy of arbitrary morphology section is written using the integral relations from [11]

$$\begin{aligned} \frac{(k_{2jil}^2 - k_{2j}^2)}{2\pi} \iint_{SCI} u_{xl}(M) G_{ss,j}(M, M^0) d\tau_M + u_{x(l-1)}^0(M^0) &= u_{xl}(M^0), \quad M^0 \in S_{Cl}, \\ \frac{\mu_{jil}(k_{2jil}^2 - k_{2j}^2)}{\mu(M^0)2\pi} \iint_{SCI} u_{xl}(M) G_{ss,j}(M, M^0) d\tau_M + u_{x(l-1)}^0(M^0) &= u_{xl}(M^0), \quad M^0 \notin S_{Cl}, \end{aligned} \quad (2)$$

where $G_{ss,j}(M, M^0)$ —function of the seismic field source, coincides with the Green function written in [11] for the corresponding problem, $k_{2jil}^2 = \omega^2(\sigma_{jil} / \mu_{jil})$ $\mu_{jil} = \mu_{ja}$ —S-wave number; u_{xl} —component of the displacement vector; u_{xl}^0 —component of the displacement vector of seismic field in the layered medium when there is no nonuniformity of the previous rank: $u_{xl}^0 = u_{x(l-1)}$ when $l = 2 \dots L$ and $u_{xl}^0 = u_x^0$ when $l = 1$, which agrees with the corresponding expression for the normal field in [11]. It is noteworthy that the structure of the equations (1) conforms with the general case when the hierarchical nonuniformity has both density and elastic characteristics different from the enclosing layer at all ranks. The only difference of the problem is the wave number values. This, the S-wave-related response is more sensitive to the nonuniform density domain in a medium. This should be taken into account in the analysis of a complex-structure geo-medium.

4. Modeling sound diffraction on 2D hierarchical nonuniformity of anomalous stress state in an N -layer elastic medium

The work [11] describes the modeling algorithm for the sound diffraction in on 2D elastic hierarchical inclusion in a J -th layer of an N -layer medium. $G_{sp,j}(M, M^0)$ —function of seismic field source, the boundary problem for this function is formulated in [12], $k_{1ji}^2 = \omega^2(\sigma_{ji} / \lambda_{ji})$ —P-wave number. Let the density of the hierarchical inclusion at all ranks l be the same as the density of the enclosing layer, while the elastic characteristics of the hierarchical inclusion differ at all ranks from the elastic characteristics of the enclosing medium; then the system of equations (3) is rewritten as:

$$\begin{aligned} \frac{(k_{1jil}^2 - k_{1j}^2)}{2\pi} \iint_{SCI} \varphi_l(M) G_{sp,j}(M, M^0) d\tau_M + \varphi_{l-1}^0(M^0) &= \varphi_l(M^0), \quad M^0 \in S_{Cl}, \\ \frac{\sigma_{jil}(k_{1jil}^2 - k_{1j}^2)}{\sigma(M^0)2\pi} \iint_{SCI} \varphi_l(M) G_{sp,j}(M, M^0) d\tau_M + \varphi_{l-1}^0(M^0) &= \varphi_l(M^0), \quad M^0 \notin S_{Cl}. \end{aligned} \quad (3)$$

The notation is the same as in the system of equations (1).

5. Modeling diffraction of elastic S-wave on hierarchical nonuniformity of anomalous stress state in an N -layer elastic medium

Similarly to (3) the algorithm for modeling an elastic S-wave propagation in an N -layer medium with a two-dimensional hierarchical structure of an arbitrary morphology section is written using the relations from [11]:

$$\begin{aligned}
& \frac{(k_{2jil}^2 - k_{2j}^2)}{2\pi} \iint_{SCI} u_{xl}(M) G_{ss,j}(M, M^0) d\tau_M + \frac{\mu_{ja}}{\mu_{jil}} u_{x(l-1)}^0(M^0) + \\
& + \frac{(\mu_{ja} - \mu_{jil})}{\mu_{jil} 2\pi} \oint_{Cl} u_{xl}(M) \frac{\partial G_{ss,j}}{\partial n} dc = u_{xl}(M^0), \quad M^0 \in S_{Cl}, \\
& \frac{\mu_{jil}(k_{2jil}^2 - k_{2j}^2)}{\mu(M^0) 2\pi} \iint_{SCI} u_{xl}(M) G_{ss,j}(M, M^0) d\tau_M + u_{x(l-1)}^0(M^0) + \\
& + \frac{(\mu_{ja} - \mu_{jil})}{\mu(M^0) 2\pi} \oint_{Cl} u_{xl}(M) \frac{\partial G_{ss,j}}{\partial n} dc = u_{xl}(M^0), \quad M^0 \in S_{Cl}.
\end{aligned} \tag{4}$$

The notation is the same as in the system of equations (2). The structure of the equations (4) coincides with the general case when both the elastic characteristics and density of the hierarchical inclusion differ at all ranks from the elastic characteristics and density of the enclosing layer. The difference of this problem is the wave number values. In this manner, the P-wave-related response of the medium is more sensitive to the elastic nonuniformity inclusion. This should be taken into account in the analysis of a complex-structure geo-medium.

Conclusion

The comparison of the expressions (1) and (2), (3) and (4) allows some conclusions to be drawn. In the integrated seismic-gravitational model disregarding anomalous stress state inside the inclusion, the analysis of the anomalous acoustic effect based on the P-wave propagation data shows its higher sensitivity relative to the shape of an inclusion as compared with the acoustic effect of S-wave propagation. It follows from the presented expressions that elastic characteristics of an enclosing medium cannot be neglected in the seismic model as these characteristics have influence on the values of anomalous density during interpretation. Otherwise the gravitational model will not reflect material constitution of a medium under analysis. In the anomalously stress state modeling without regard to anomalous influence of density nonuniformity inside the inclusions, the analysis of the anomalous acoustic effect using the S-wave propagation data shows its higher sensitivity as compared to the acoustic effect of P-wave propagation. However, the density of enclosing medium should not be disregarded in the seismic model as the density values influence the interpretation of values of the anomalous elastic characteristics generating the anomalous stress state. Otherwise, the geomechanical model will not reflect adequately stress state of the analyzed medium.

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